

Show your work for full credits.

1. Evaluate

$$\begin{bmatrix} 2 & -1 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$$

2. Evaluate

$$\begin{bmatrix} 2 & -1 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$$

3. Express the system of equations in column form and matrix form.

$$\begin{aligned} 2x - y &= 4 \\ 3x + 2y &= -2 \end{aligned}$$

4. Is the given a singular case? Justify

$$\begin{aligned} 3x - y &= 7 \\ -6x + 2y &= 5 \end{aligned}$$

5. Find the values of x , y , and z for the given equations by elimination method.

$$\begin{aligned} x + y - z &= -4 \\ 3x - y + z &= 8 \\ -2x + 3y - 2z &= -14 \end{aligned}$$

6. the inverse of the given matrix. Then, show that product of the given matrix and the inverse is the identity.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

7. If the given matrix is singular, find a value for a .

$$\begin{bmatrix} 2 & a \\ a & 8 \end{bmatrix}$$

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Part I – (95 pts)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

1. Find the inverse of the given matrix, A .
2. Factor A into LU .
3. Find the transpose of A .

4. Factor B into LU , where $a, b, c > 0$. (5 pts)

$$B = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$

5. A is 2×2 matrix with $A^6 = I$. Find A .

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Total: 105 points

Part I – 95 pts

1) Addition and scalar multiplication are required to satisfy these eight rules:

1. $x + y = y + x$.
2. $x + (y + z) = (x + y) + z$.
3. There is a unique “zero vector” such that $x + 0 = x$ for all x .
4. For each x there is a unique vector $-x$ such that $x + (-x) = 0$.
5. $1x = x$.
6. $(c_1 c_2)x = c_1(c_2 x)$.
7. $c(x + y) = cx + cy$.
8. $(c_1 + c_2)x = c_1 x + c_2 x$.

Suppose $(x_1, x_2) + (y_1, y_2)$ is defined to be $(x_1 + y_1, x_2 + y_1)$. With the usual multiplication $c\mathbf{x} = (cx_1, cx_2)$, which of the eight conditions are not satisfied? (Write at least one rule and explain.)

2) Describe the column space and the null space for the given matrix.

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

3) In \mathbf{R}^2 , explain why a set of $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ can be a subspace but not a set of

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

Part II – 10 pts

4) Construct a matrix whose null space consists of all linear combinations of

$$\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 5 \\ 0 \\ 1 \end{bmatrix}. \text{ (pivot columns 1 and 2, free variables } x_3 \text{ and } x_4)$$

Quarter 1 Exam
Linear Algebra

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Part I – 95 points

1. Find the column space and the null space for

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 2 \end{bmatrix}$$

2. Find the values of x, y , and z for the given equations.

$$\begin{aligned} x + y + z &= 3 \\ 2x + 3y + 4z &= 11 \\ 4x + 3y + z &= 5 \end{aligned}$$

3. Find LU factorization for B.

$$B = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

Part II – 10 points

4. Find a 2×2 matrix, where inverse of the matrix is the same as the transpose of the matrix. (The matrix cannot contain 0 or 1 as an element.)